Engineering Notes

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Application of Oscillatory Aerodynamic Theory to Estimation of Dynamic Stability Derivatives

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WHILE the development of practical, three-dimensional, lifting-surface theories[1–4] for flutter analysis throughout the various Mach number regimes was still in its infancy, Etkin[5] (pp. 161–164) pointed out the usefulness of the theory of oscillating wings for calculating dynamic stability derivatives. Etkin demonstrated the calculation using the two-dimensional solution of Theodorsen,[6] presumably because he was not in a position to make three-dimensional calculations. He immediately ran into the difficulty that the two-dimensional solution does not possess a Maclaurin series in the reduced frequency since the derivative of the imaginary part of Theodorsen's function has a logarithmic singularity at zero frequency. Although Etkin emphasized that this difficulty did not arise in the three-dimensional case, perhaps the unfortunate behavior of the two-dimensional solution has tended to discourage many Stability and Control Engineers from following the progress in oscillatory aerodynamic theory that has occurred since publication of Etkin's book (1959). (The appearance of a new aircraft stability handbook[7] seems to support this opinion.) Surveys of this progress have been presented in 1962 by Rodden and Revell,[8] in 1965 by Ashley, Widnall, and Landahl,[9] and in 1968 by Landahl and Stark.[10] It is the purpose of this Note to restate Etkin's case for use of oscillatory aerodynamic theory in the context of the current state-of-the-art of unsteady lifting-surface theory and to extend his method to obtain higher-order derivatives.

Before considering the calculation of dynamic stability derivatives, it may be well to place the concept of dynamic stability derivatives in perspective. Quoting Etkin[5] (pp. 122–123), "The method of treating the aerodynamic force and moment perturbations which is given below is essentially that introduced by Bryan.[11] It leads to a form for the equations of motion that has been used with great success during the past half century. Despite this success, however, the method is not (mathematically speaking) sound. Indeed it does not give correct answers in certain cases where the aerodynamic forces change very rapidly as in the penetration of a sharp-edged gust at high speed, or when a control is very rapidly displaced." The basic limitation of the method of Bryan is its inability to account for the lagging mechanism of aerodynamic loads, i.e., the loads depend not only on the instantaneous motion but also its time history; Bryan's method has no "memory." In the case of oscillatory motion, which is the subject of this paper, these limitations appear in the form of phase shifts. Alternative methods have been proposed by Temple[12] and Etkin[13] which are not subject to these objections. However, despite the shortcomings of Bryan's method, it is obvious that it will continue to be an effective engineering tool for some time to come. With due respect for Bryan's heritage but with the limitations of his theory clearly in mind, we now address ourselves to the calculation of dynamic stability derivatives using steady and oscillatory lifting-surface theory.

It is sufficient for illustrative purposes to consider the longitudinal derivatives for pitch rate $q = \dot{\theta}$, and for rate of change of angle of attack $\dot{\alpha}$. The Maclaurin series for the longitudinal stability derivatives appears as

$$C_z = C_{z_0} + C_{z_\alpha}\alpha + C_{z_{\dot\alpha}}(\dot\alpha\bar c/2V) + C_{z_q}(\dot\theta\bar c/2V) + C_{z_{\ddot\alpha}}(\ddot\alpha\bar c^2/4V^2) + C_{z_{\dot q}}(\ddot\theta\bar c^2/4V^2) + \ldots \quad (1)$$

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\dot\alpha}}(\dot\alpha\bar c/2V) + C_{m_q}(\dot\theta\bar c/2V) + C_{m_{\ddot\alpha}}(\ddot\alpha\bar c^2/4V^2) + C_{m_{\dot q}}(\ddot\theta\bar c^2/4V^2) + \ldots \quad (2)$$

The intercept values C_{z_0} and C_{m_0} will be omitted from further consideration since they cannot be estimated from oscillatory aerodynamic theory. We note also that, although the derivatives C_{z_α}, C_{m_α}, C_{z_q}, and C_{m_q} may be estimated by oscillatory aerodynamic theory for low reduced frequency, they are static derivatives that should be determined as such. We, therefore, assume in the following that the static derivatives are known. Truncating the series in Eqs. (1) and (2) with the velocity terms, deleting C_{z_0} and C_{m_0}, and considering the case of harmonic pitching‡ at frequency ω, $\theta = \alpha = \alpha_0 Rl(e^{i\omega t})$, we find the lift and moment coefficients to be

$$C_z = Rl(\bar C_z e^{i\omega t}) \quad (3)$$

and

$$C_m = Rl(\bar c_m e^{i\omega t}) \quad (4)$$

where the complex amplitudes of the coefficients are

$$\bar C_z = \alpha_0[C_{z_\alpha} + ik(C_{z_{\dot\alpha}} + C_{z_q})] \quad (5)$$

$$\bar C_m = \alpha_0[C_{m_\alpha} + ik(C_{m_{\dot\alpha}} + C_{m_q})] \quad (6)$$

and $k = \omega\bar c/2V$ is the reduced frequency in which $\bar c$ is the reference chord and V is the freestream velocity.

We next consider deriving the complex coefficients from oscillatory aerodynamic theory. A complex matrix of frequency dependent oscillatory aerodynamic influence coefficients (AIC's) $[C_h]$ has been defined in Ref. 8 that relates the column matrix of control point forces $\{F\}$, positive downward,

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‡ Note that the motion with amplitude α_0 is not pure "pitching" but a combination of pitch and plunge (or change in flight path angle) as these motions are usually comprehended in stability and control analysis.

Table 1 **Lift and moment coefficients for pitching and plunging of a typical jet transport wing at a Mach number of 0.8**

	Pitching		Plunging	
k	$\bar{C}_z/\alpha_0$	$\bar{C}_m/\alpha_0$	$\bar{C}_z/ikh_0(2/\bar{c})$	$\bar{C}_m/ikh_0(2/\bar{c})$
0.001	−5.862 + i0.0066	−7.481 + i0.0055	−5.862 + i0.0126	−7.481 + i0.0158
0.002	−5.862 + i0.0131	−7.481 + i0.0109	−5.862 + i0.0252	−7.481 + i0.0315
0.004	−5.861 + i0.0262	−7.480 + i0.0217	−5.860 + i0.0504	−7.480 + i0.0630
0.005	−5.860 + i0.0327	−7.479 + i0.0271	−5.860 + i0.0630	−7.478 + i0.0787
0.010	−5.854 + i0.0649	−7.471 + i0.0535	−5.853 + i0.1253	−7.469 + i0.1566
0.020	−5.832 + i0.1256	−7.443 + i0.1015	−5.826 + i0.2458	−7.435 + i0.3069
0.040	−5.755 + i0.2250	−7.342 + i0.1684	−5.730 + i0.4612	−7.312 + i0.5740
0.050	−5.704 + i0.2617	−7.277 + i0.1851	−5.667 + i0.5534	−7.232 + i0.6876
0.100	−5.405 + i0.3169	−6.889 + i0.1033	−5.283 + i0.8568	−6.742 + i1.0543

to the deflections $\{h\}$, positive downward, of the same points through the equation§

$$\{F\} = \rho\omega^2(\bar{c}/2)^2 sRl([C_h]\{h\}e^{i\omega t}) \tag{7}$$

where ρ is the density and s is the reference semispan. From Eq. (7), the downward lift Z and nose-up pitching moment M about station x_0 leads to the corresponding coefficients.

$$C_z = Z/\tfrac{1}{2}\rho V^2 S \tag{8a}$$

$$C_z = Rl(\bar{C}_z e^{i\omega t}) \tag{8b}$$

where

$$\bar{C}_z = 2k^2(s/S)\lfloor I\rfloor[C_h]\{h\} \tag{9}$$

and

$$C_m = M/\tfrac{1}{2}\rho V^2 S\bar{c} \tag{10a}$$

$$C_m = Rl(\bar{C}_m e^{i\omega t}) \tag{10b}$$

where

$$\bar{C}_m = 2k^2(s/S\bar{c})\lfloor x_0 - x\rfloor[C_h]\{h\} \tag{11}$$

In the preceding $\lfloor I\rfloor$ and $\lfloor x_0 - x\rfloor$ denote row matrices of unity and lever arms, respectively, and S is the reference area.

To obtain $C_{z_{\dot{\alpha}}}$ and $C_{m_{\dot{\alpha}}}$, it is necessary to evaluate Eqs. (9) and (11) for pitching motion at some reduced frequency k. For pitching about station x_1 with amplitude α_0, the deflection mode is $\{h\} = \alpha_0\{x_1 - x\}$. By identifying the resulting *imaginary* parts of Eqs. (9) and (11) at the same reduced frequency with Eqs. (5) and (6) the derivatives $C_{z_{\dot{\alpha}}}$ and $C_{m_{\dot{\alpha}}}$ follow since C_{z_q} and C_{m_q} are already known. The resulting values are somewhat dependent on the choice of reduced frequency k. Strictly speaking, the dynamic stability derivatives are defined by limiting values as k approaches zero. A small value of k may be chosen for the calculation, but large enough not to create numerical round-off errors, to obtain reliable results. An alternate approach is to choose k large enough to cover the range of frequency of interest, say, in autopilot studies. In this manner, the dynamic derivatives become representative of the frequencies of interest rather than just at the limit of zero frequency. If the range of frequency is very large, it may be necessary to include higher-order dynamic stability derivatives as shown in Eqs. (1) and (2). In this case, truncating the series with the acceleration terms and again considering harmonic pitching, we find

$$\bar{C}_z/\alpha_0 = C_{z_\alpha} + ik(C_{z_{\dot{\alpha}}} + C_{z_q}) - k^2(C_{z_{\ddot{\alpha}}} + C_{z_{\dot{q}}}) \tag{12}$$

$$\bar{C}_m/\alpha_0 = C_{m_\alpha} + ik(C_{m_{\dot{\alpha}}} + C_{m_q}) - k^2(C_{m_{\ddot{\alpha}}} + C_{m_{\dot{q}}}) \tag{13}$$

We observe here that the acceleration terms appear only as their sum so it becomes necessary also to consider harmonic plunging with amplitude h_0 [$\theta = 0$, $\alpha = \dot{h}/V = (h_0/V)Rl(i\omega e^{i\omega t})$]. For harmonic plunging, we find

$$\bar{C}_z/h_0(2/\bar{c}) = ikC_{z_\alpha} - k^2C_{z_{\dot{\alpha}}} - ik^3C_{z_{\ddot{\alpha}}} \tag{14}$$

$$\bar{C}_m/h_0(2/\bar{c}) = ikC_{m_\alpha} - k^2C_{m_{\dot{\alpha}}} - ik^3C_{m_{\ddot{\alpha}}} \tag{15}$$

Evaluating Eqs. (9) and (11) for the plunging motion $\{h\} = h_0\{I\}$ at the reduced frequency k leads to $C_{z_{\ddot{\alpha}}}$ and $C_{m_{\ddot{\alpha}}}$ from the *imaginary* parts since C_{z_α} and C_{m_α} are known. Then from the previous evaluation of Eqs. (9) and (11) for the pitching motion, the *real* parts lead to $C_{z_{\dot{q}}}$ and $C_{m_{\dot{q}}}$ in Eqs. (12) and (13) since all of the other derivatives are already known.

As an example of the previous procedure, we consider a typical jet transport wing having an aspect ratio of 8.0, a taper ratio of 0.25, a quarter-chord sweep of 30°, and flying at a Mach number of 0.8. The moment coefficient is referred to the pitching axis location at the quarter-chord point of the mean aerodynamic chord. Table 1 summarizes the complex lift and moment coefficients for 9 values of reduced frequency in the range 0.001–0.100. The calculations were carried out using the oscillatory lifting-surface method described in Ref. 14 which is based on the Doublet Lattice Method of Albano and Rodden.[15] The static derivatives were calculated by a steady lifting-surface method described in Ref. 16 which is based on the Vortex Lattice Method of Hedman.[17] The static derivatives were found to be $C_{z_\alpha} = -5.862$, $C_{m_\alpha} = -7.482$, $C_{z_q} = -6.050$, and $C_{m_q} = -10.323$. Table 2 presents values of $C_{z_{\dot{\alpha}}}$ based on Eqs. (5) and (9) and $C_{m_{\dot{\alpha}}}$ based on Eqs. (6) and (11) calculated in the sequence outlined previously; the Table also presents the values of the acceleration derivatives $C_{z_{\ddot{\alpha}}}$, $C_{m_{\ddot{\alpha}}}$, $C_{z_{\dot{q}}}$, and $C_{m_{\dot{q}}}$ based on Eqs. (9), (11), and (12–15) computed as outlined previously. Some dependence of the coefficients on the chosen value of frequency is evident in the Tables. The frequency dependence is particularly evident in the coefficients $C_{z_{\ddot{\alpha}}}$ and $C_{m_{\ddot{\alpha}}}$ at low reduced frequencies. However, practical use of the results can be made by selecting values of the derivatives in the frequency range representative of the aircraft maneuvering or autopilot characteristics. In this sense we have a variation on the least-squares method of Goland[18] which he applied to approximate the damping of a two-dimensional airfoil with a control surface in an incompressible flow.

A procedure may be followed to reduce the frequency dependence of the damping and acceleration derivatives by delaying truncation of the series of Eqs. (1) and (2) to a point where the coefficients may be based on two reduced frequencies k_1 and k_2 rather than just the one required by the previous technique. If we extend Eq. (1) to include the terms $C_{z_{\dddot{\alpha}}}(\dddot{\alpha}\bar{c}^3/8V^3) + C_{z_{\ddot{q}}}(\dddot{\theta}\bar{c}^3/8V^3) + C_{z_{\alpha^{IV}}}(\alpha^{IV}\bar{c}^4/16V^4) +$

§ In connection with the estimation of the intercept values C_{z_0} and C_{m_0} and the static derivatives C_{z_α}, C_{m_α}, C_{z_q}, C_{m_q}, Ref. 8 has defined static AIC's by the equation $\{F\} = (qS/\bar{c})[C_{hs}]\{h\}$. The α derivatives are determined by a linear streamwise (x direction, positive forward) variation in deflection from the point of rotation x_1, i.e., $\{h\} = \alpha\{x_1 - x\}$. The q derivatives are determined from an effective parabolic camber so the deflection has a quadratic streamwise variation, i.e., $\{h\} = q\{(x_1 - x)^2\}$.

Table 2 Estimates of $C_{z_{\dot{\alpha}}}$, $C_{m_{\dot{\alpha}}}$, $C_{z_{\ddot{\alpha}}}$, $C_{m_{\ddot{\alpha}}}$, $C_{z_{\dot{q}}}$, and $C_{m_{\dot{q}}}$ based on a single value of k

k	$C_{z_{\dot{\alpha}}}$	$C_{m_{\dot{\alpha}}}$	$C_{z_{\ddot{\alpha}}}$	$C_{m_{\ddot{\alpha}}}$	$C_{z_{\dot{q}}}$	$C_{m_{\dot{q}}}$
0.001	12.62	15.77	173.0	226.0	−16.5	−20.2
0.002	12.62	15.77	113.8	145.8	−16.5	−20.2
0.004	12.60	15.72	100.9	129.6	−16.5	−20.2
0.005	12.59	15.72	98.8	127.0	−16.5	−20.2
0.010	12.53	15.66	94.7	122.5	−16.4	−20.2
0.020	12.29	15.34	90.9	117.4	−15.8	−19.4
0.040	11.52	14.30	82.4	105.9	−15.2	−18.8
0.050	11.07	13.70	77.9	100.0	−14.5	−18.5
0.100	8.57	10.54	57.9	74.0	−12.1	−14.6

$C_{z_{\dddot{q}}}(\theta^{IV}\bar{c}^4/16V^4)$, and Eq. (2) to include the terms $C_{m_{\dddot{\alpha}}}(\dddot{\alpha}\bar{c}^3/8V^3) + C_{m_{\ddot{q}}}(\dddot{\theta}\bar{c}^3/8V^3) + C_{m_{\alpha^{IV}}}(\alpha^{IV}\bar{c}^4/16V^4) + C_{m_{\dddot{q}}}(\theta^{IV}\bar{c}^4/16V^4)$, then Eqs. (12) and (13) become

$$\bar{C}_z(k)/\alpha_0 = C_{z_\alpha} + ik(C_{z_{\dot{\alpha}}} + C_{z_q}) - k^2(C_{z_{\ddot{\alpha}}} + C_{z_{\dot{q}}}) - ik^3(C_{z_{\dddot{\alpha}}} + C_{z_{\ddot{q}}}) + k^4(C_{z_{\alpha^{IV}}} + C_{z_{\dddot{q}}}) \quad (16)$$

$$\bar{C}_m(k)/\alpha_0 = C_{m_\alpha} + ik(C_{m_{\dot{\alpha}}} + C_{m_q}) - k^2(C_{m_{\ddot{\alpha}}} + C_{m_{\dot{q}}}) - ik^3(C_{m_{\dddot{\alpha}}} + C_{m_{\ddot{q}}}) + k^4(C_{m_{\alpha^{IV}}} + C_{m_{\dddot{q}}}) \quad (17)$$

and Eqs. (14) and (15) become

$$\bar{C}_z(k)/h_0(2/\bar{c}) = ikC_{z_\alpha} - k^2C_{z_{\dot{\alpha}}} - ik^3C_{z_{\ddot{\alpha}}} + k^4C_{z_{\dddot{\alpha}}} + ik^5C_{z_{\alpha^{IV}}} \quad (18)$$

$$\bar{C}_m(k)/h_0(2/\bar{c}) = ikC_{m_\alpha} - k^2C_{m_{\dot{\alpha}}} - ik^3C_{m_{\ddot{\alpha}}} + k^4C_{m_{\dddot{\alpha}}} + ik^5C_{m_{\alpha^{IV}}} \quad (19)$$

Evaluating Eqs. (9) and (11) for both pitching and plunging motions at both reduced frequencies k_1 and k_2 and identifying the *real* and *imaginary* parts with their correspondents in Eqs. (16–19) leads to a sufficient number of simultaneous equations so that *all* of the dynamic stability derivatives can be determined. Although the higher-order stability derivatives may be required by, say, the frequency characteristics of an automatic control system,¶ for the present, they are only regarded as a means to an end, i.e., of refining the velocity and acceleration derivatives, and are not shown. The refined estimates of the velocity and acceleration derivatives are shown in Table 3 for various pairs of representative reduced frequencies. Again, a frequency dependence is evident but less so than that observed in Table 2 for all derivatives except, as before, $C_{z_{\ddot{\alpha}}}$ and $C_{m_{\ddot{\alpha}}}$. Thus, the selection of representative values is more convenient. Comparing Tables 2 and 3, we note that the average values in Table 3 are larger in magnitude than their counterparts in Table 2. The reason is that the higher order derivatives have been separated out in Table 3, whereas they are implicitly present in Table 2.

Table 3 Estimates of $C_{z_{\dot{\alpha}}}$, $C_{m_{\dot{\alpha}}}$, $C_{z_{\ddot{\alpha}}}$, $C_{m_{\ddot{\alpha}}}$, $C_{z_{\dot{q}}}$, and $C_{m_{\dot{q}}}$ based on two values of k

k_1,k_2	$C_{z_{\dot{\alpha}}}$	$C_{m_{\dot{\alpha}}}$	$C_{z_{\ddot{\alpha}}}$	$C_{m_{\ddot{\alpha}}}$	$C_{z_{\dot{q}}}$	$C_{m_{\dot{q}}}$
0.001,0.002	12.62	15.77	192.8	255.0	−16.5	−20.2
0.002,0.004	12.62	15.77	118.0	163.3	−16.5	−20.2
0.005,0.010	12.62	15.77	100.2	128.8	−16.5	−20.2
0.010,0.020	12.61	15.77	96.0	124.2	−16.5	−20.2
0.020,0.040	12.55	15.68	93.8	121.2	−16.4	−20.0
0.050,0.100	11.90	14.82	84.6	108.6	−15.7	−19.2

¶ It should be noted that their inclusion in the equations of motion will result in increasing the order of the system of differential equations.

The behavior of $C_{z_{\ddot{\alpha}}}$ and $C_{m_{\ddot{\alpha}}}$ as k approaches zero is singular like k^{-1} since their evaluation is equivalent to, e.g.,

$$C_{z_{\ddot{\alpha}}} = \lim_{k\to 0} Rl[(d^2/dk^2)(\bar{C}_z/\alpha_0) + (1/k)(d/dk)(\bar{C}_z/\alpha_0)] \quad (20)$$

If the first derivative is finite, the singular behavior appears and reflects the limitation of Bryan's method; higher-order stability derivatives will possess higher-order singularities. It should not be inferred from this limitation that C_z and C_m are not analytic functions of k on three-dimensional wings since Miles[19] has indicated that they probably are analytic. The limitation is clearly seen in Eqs. (18) and (19) wherein the real parts of $\bar{C}_z$ and $\bar{C}_m$ must be expressed as a power series in k using only even powers of k, while the power series for the imaginary parts uses only odd powers. The oscillatory aerodynamic theory, being completely general, has no such restriction, and both real and imaginary parts can be expressed as a power series in k using both even and odd powers.** The singular behavior is apparent for $C_{z_{\ddot{\alpha}}}$ and $C_{m_{\ddot{\alpha}}}$ in Tables 2 and 3 for small values of k. The singularity, however, is very localized and shows itself only at very small values of k. This indicates that the singularity is very weak, i.e., the coefficient of $1/k$ is very small. For the case of two-dimensional flow, the coefficient of the $1/k$ term (related to the real part of the Theodorsen function) is by no means small. However, it is encouraging to note that, even for the high aspect ratio wing considered, this coefficient remains small. As before, practical use of the results can be made by selecting values of the derivatives representative of the frequency range of interest.

In the final analysis, however, perhaps the only mathematically rigorous approach to the inclusion of three-dimensional oscillatory aerodynamic theory is to adapt Fourier Transform techniques to meet the requirements of Stability and Control Analyses since oscillatory aerodynamic solutions are to be regarded as Fourier Transforms of transient aerodynamic solutions. The development of the "Fast Fourier Transform" technique by Cooley and Tukey[20] makes such an approach computationally feasible. The procedure would be to determine the frequency response of the vehicle to a harmonic excitation, e.g., stick motion or control surface motion. The inverse Fourier transform of the frequency response is then the response time history to an impulsive input. From the time history, the stability of the various response modes, e.g., phugoid or Dutch roll, may be estimated. For arbitrary inputs (other than impulsive), the response may be found by convolution of the system frequency response and the input frequency response.

It was considered sufficient to illustrate the use of unsteady AIC's for the calculation of longitudinal stability derivatives. Similarly, it should be apparent that many of the lateral-directional dynamic stability derivatives can also be found from oscillatory aerodynamic theory. However, theoretical prediction of stability derivatives for complete aircraft is not yet within the state of the art. This is so in the longitudinal case because unsteady wing-fuselage-tail interference theory has only recently attracted serious investigation. We note, however, that the Doublet Lattice Method, applied here to a planar wing, can be applied in its nonplanar form to wing-fuselage-tail interference if the fuselage is idealized as an annular wing; the validity of the idealization remains to be seen. In the lateral-directional case, theoretical prediction methods are somewhat limited

** A possible approach to reducing the incompatibility might be to disregard the terms in the series for the oscillatory coefficients that have no counterpart in Bryan's method, i.e., to disregard the odd powered terms in the real part and the even powered terms in the imaginary part.

again because of fuselage interference, but also because the problem of unsteady induced drag has not yet been solved in general. Belotserkovskii[21] (pp. 29–33) has considered the Kutta-Joukowsky Theorem "in the small" for unsteady flow. As such, it can only be used for prediction of normal components of aerodynamic loads, and cannot be used to obtain the distribution of the so-called leading edge suction force. The unsteady suction forces are an essential ingredient in the calculation of rotary derivatives. Much work remains to be done, but we cannot agree with the pessimistic evaluation of the state of the art for determining stability derivatives given in Ref. 7, but then—we do not restrict the use of oscillatory aerodynamic theory to flutter analysis.

References

[1] Watkins, C. E., Woolston, D. S., and Cunningham, H. J., "A Systematic Kernel Function Procedure for Determining Aerodynamic Forces on Oscillating or Steady Finite Wings at Subsonic Speeds," Rept. R-48, 1959, NASA.

[2] Pines, S., Dugundji, J., and Neuringer, J., "Aerodynamic Flutter Derivatives for a Flexible Wing with Supersonic and Subsonic Edges," *Journal of the Aerospace Sciences*, Vol. 22, No. 10, Oct. 1955, pp. 693–700.

[3] Rodemich, E. R. and Andrew, L. V., "Unsteady Aerodynamics for Advanced Configurations; Part II—A Transonic Box Method for Planar Lifting Surfaces," FDL-TDR-64-152, Pt. II, May 1965, U.S. Air Force Flight Dynamics Lab., Wright-Patterson Air Force Base, Ohio.

[4] Ashley, H. and Zartarian, G., "Piston Theory—A New Aerodynamic Tool for the Aeroelastician," *Journal of the Aerospace Sciences*, Vol. 23, No. 12, Dec. 1956, pp. 1109–1118.

[5] Etkin, B., *Dynamics of Flight*, Wiley, New York, 1959.

[6] Theodorsen, T., "General Theory of Aerodynamic Instability and the Mechanism of Flutter," Rept. 496, 1935, NACA.

[7] "An Analysis of Methods for Predicting the Stability Characteristics of an Elastic Airplane—Summary Report," CR-73277, Nov. 1968, NASA.

[8] Rodden, W. P. and Revell, J. D., "Status of Unsteady Aerodynamic Influence Coefficients," Paper FF-33, 1962, Institute of the Aeronautical Sciences; preprinted as Rept. TDR-930-(2230-09)TN-2, 1961, Aerospace Corp.

[9] Ashley, H., Widnall, S., and Landahl, M. T., "New Directions in Lifting Surface Theory," *AIAA Journal*, Vol. 3, No. 1, Jan. 1965, pp. 3–16.

[10] Landahl, M. T. and Stark, V. J. E., "Numerical Lifting-Surface Theory—Problems and Progress," *AIAA Journal*, Vol. 6, No. 11, Nov. 1968, pp. 2049–2060.

[11] Bryan, G. H., *Stability in Aviation*, Macmillan, London, 1911.

[12] Temple, G., "The Representation of Aerodynamic Derivatives," R and M No. 2114, 1945, Aeronautical Research Council, London.

[13] Etkin, B., "Aerodynamic Transfer Functions: An Improvement on Stability Derivatives for Unsteady Flight," UTIA Rept. 42, 1956, Univ. of Toronto.

[14] Stahl, B. et al., "Aerodynamic Influence Coefficients for Oscillating Planar Lifting Surfaces by the Doublet Lattice Method for Subsonic Flows Including Quasi-Steady Fuselage Interference," Rept. DAC-67201, Oct. 1968, McDonnell Douglas Corp.

[15] Albano, E. and Rodden, W. P., "A Doublet-Lattice Method for Calculating Lift Distributions on Oscillating Surfaces in Subsonic Flows," *AIAA Journal*, Vol. 7, No. 2, Feb. 1969, pp. 279–285; "errata," *AIAA Journal*, Vol. 7, No. 11, Nov. 1969, p. 2192.

[16] Giesing, J. P., "Lifting Surface Theory for Wing-Fuselage Combinations," Rept. DAC-67212, Aug. 1, 1968, McDonnell Douglas Corp.

[17] Hedman, S. G., "Vortex Lattice Method for Calculation of Quasi-Steady State Loadings on Elastic Wings in Subsonic Flow," Rept. FFA 105, Oct. 1965, Aeronautical Research Institute of Sweden, Stockholm.

[18] Goland, M., "The Quasi-Steady Air Forces for Use in Low-Frequency Stability Calculations," *Journal of the Aerospace Sciences*, Vol. 17, No. 10, Oct. 1950, pp. 601–608 and 672.

[19] Miles, J. W., "Unsteady Flow Theory in Dynamic Stability," *Journal of the Aerospace Sciences*, Vol. 17, No. 1, Jan. 1950, p. 62.

[20] Cooley, J. W. and Tukey, J. W., "An Algorithm for the Machine Computation of Complex Fourier Series," *Mathematics of Computation*, Vol. 19, No. 90, 1965, pp. 297–301.

[21] Belotserkovskii, S. M., *The Theory of Thin Wings in Subsonic Flow*, Plenum Press, New York, 1967.

Starting Criterion for Hypersonic Inlets

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THE starting criterion for supersonic diffusers, which assumes that the maximum contraction ratio can be calculated solely from normal shock losses, is not valid in some hypersonic applications. This invalidity has been demonstrated in wind-tunnel tests of an early design hypersonic scramjet inlet with a large turbulent boundary layer relative to the inlet height.[1] The purpose of this Note is to present experimental results that show quantitatively when the assumption of only normal shock losses in the starting criteria is adequate.

A one-dimensional adiabatic flow analysis,[2] assuming negligible back pressure, shows that for a supersonic wind tunnel incorporating a second minimum or for a fixed geometry inlet (Fig. 1a) starting is a function of total pressure recovery $p_{t,2}/p_{t,1}$, contraction ratio A_1/A_2, and Mach number M_1. Since the maximum contraction ratio for starting is obtained when the flow at the second minimum is sonic, the previous parameters can be expressed by the following equa-

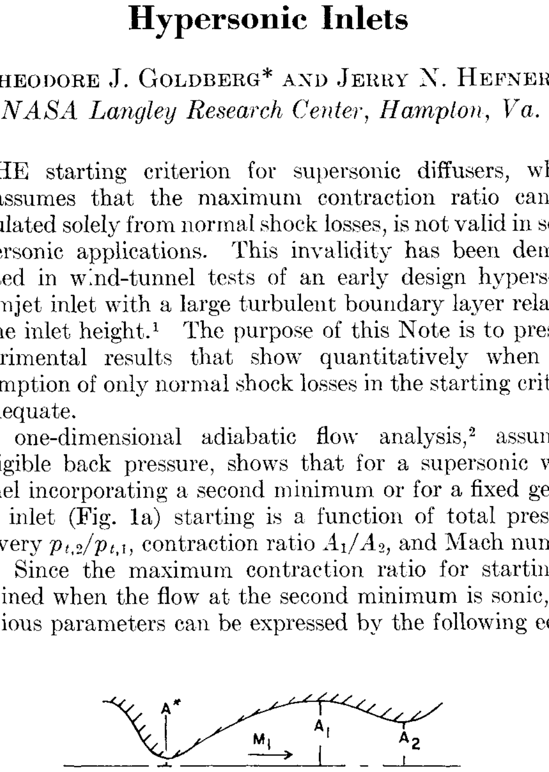

Fig. 1. Starting parameters for hypersonic diffusers.

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